Estimating an Equilibrium Model

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Motivation

- In low-income countries, schooling does not equal learning.
  - In rural Pakistan, 25% of students do not experience test score gains from year to year.
  - Policies focus on improving "vertical" school quality (teacher training, improved inputs) and have small or moderate effects.
- Mismatch between instruction level and student ability is important.
  - Particularly true in low-income countries.
- Know little about whether misallocation of instructional level affects learning and how it responds to market structure.
  - Private schooling share is high in low-income countries.

This Paper I

- To motivate the importance of instructional match, provide evidence that private school entry increases *within-school* inequality in test scores in private schools.
- Develop and estimate a novel model in which schools choose instruction level and vertical quality.
- Profit-maximizing private schools take into account different students' differential responsiveness to quality leading to a Spence Distortion.
- Apply this general framework to education markets in Pakistan.

This Paper II

- Estimate the distribution of private schools' instruction levels.
  - The average private school strongly caters to more advantaged students.
- Provides a structural estimate of the importance of match-specific quality.
  - Moving a student from their worst to best match school increases test scores by ≈ 1 year of test score gains.
- Model predicts that private school entry leads to greater inequality between more and less disadvantaged students' learning *within* a school.
  - Confirm this is the case in difference-in-differences regressions.
- Calculate partial equilibrium counterfactuals where instructional levels are set to maximize learning.
  - Greatly reduces inequality in learning and moderately increases average learning.
  - Low responsiveness to quality in enrollment decisions limits gains in counterfactuals.

Context: Pakistan

- In rural Pakistan, 35% of students are enrolled in unregulated private schools.
- The typical school has 1 or less teachers per grade.
- The mean private school in the data costs 5% of per capita annual income.
- Villages are closed educational markets. $\Rightarrow$ Why is this important?
- Average village has ≈ 3 private schools.

Data

- LEAPS Pakistan data consists of a random sample of 112 villages in the province of Punjab.
- Three key surveys (each conducted every year from 2004-2007):
  - Geo-coded survey of universe of schools.
  - Survey of children in the schools (**child survey**), including low-stakes test scores.
  - Geo-coded survey of a sub-sample of households in the village (**household survey**).
  - Importantly, some overlap in the household and child survey.

Market Structure and Test Scores

To provide evidence that school quality is different for different students,

- Estimate the effects of private school entry on the average student's test scores in public and private schools.
- Then, estimate the effect of private school entry on inequality in test scores in the public and private sector.

Drivers of Exit and Entry

Effect of Competition on Average Test Scores

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Math		English		Urdu		Mean	
	Private	Public	Private	Public	Private	Public	Private	Public
num_pri_{vt}	-0.078**	0.006	-0.039	-0.010	-0.010	-0.031	-0.038	-0.012
	(0.031)	(0.045)	(0.046)	(0.037)	(0.036)	(0.030)	(0.034)	(0.036)
Num Gov. School Control	Y	Y	Y	Y	Y	Y	Y	Y
Report Card-Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Lagged Test Scores	Y	Y	Y	Y	Y	Y	Y	Y
Grade FE	Y	Y	Y	Y	Y	Y	Y	Y
Village FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	6,788	13,938	6,788	13,938	6,788	13,938	6,788	13,938
Clusters	108	112	108	112	108	112	108	112
Adjusted R^2	0.534	0.565	0.550	0.571	0.611	0.606	0.655	0.671

No effect on the test scores of the average student in either sector.

Effect of Competition on Inequality

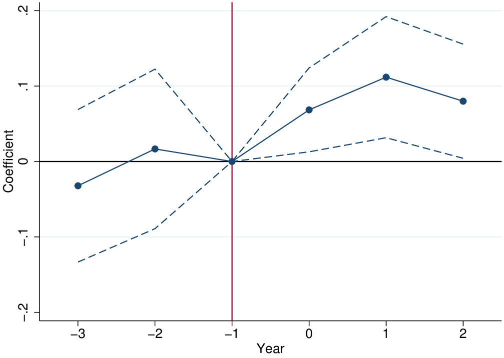
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Variance Math		Variance English		Variance Urdu		Variance Mean	
	Private	Public	Private	Public	Private	Public	Private	Public
num_pri_{vt}	0.085***	0.022	0.096***	0.047	0.098***	0.043	0.077***	0.034
	(0.026)	(0.025)	(0.032)	(0.031)	(0.033)	(0.030)	(0.023)	(0.025)
Num. Gov. School Control	Y	Y	Y	Y	Y	Y	Y	Y
Report Card-Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Village FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	426	447	426	447	426	447	426	447
Clusters	108	112	108	112	108	112	108	112
Adjusted R^2	0.292	0.400	0.248	0.362	0.242	0.311	0.322	0.412

Note: variance is calculated at the village-year level.

- Null effect on the average student masks significant heterogeneous effects in the private sector.

Effect of Competition on Inequality

Figure: Effect of Entry on Village-Level Variance in Private-Sector Test Scores



$$inequality_{vt} = \rho_0 + \gamma_1 event_{v,s-3} + \gamma_2 event_{v,s-2} + \gamma_3 event_{v,0} + \gamma_4 event_{v,s+1} + \gamma_5 event_{v,s+2} + \alpha_t + \psi_v + \Gamma \mathbf{X_{vt}} + \epsilon_{vt}$$

What Drives the Increase in Private Sector Inequality?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Variance Math		Variance English		Variance Urdu		Variance Mean	
	No Switchers	School-Level Variance	No Switchers	School-Level Variance	No Switchers	School-Level Variance	No Switchers	School-Level Variance
num_pri_{vt}	0.099**	0.089***	0.078*	0.063**	0.130**	0.049	0.089***	0.060**
	(0.038)	(0.032)	(0.041)	(0.027)	(0.050)	(0.036)	(0.033)	(0.024)
Num Gov. School Control	Y	Y	Y	Y	Y	Y	Y	Y
Report Card-Year FE	Y	Y	Y	Y	Y	Y	Y	Y
School FE	N	Y	N	Y	N	Y	N	Y
Village FE	Y	N	Y	N	Y	N	Y	N
Peer Controls	Y	N	Y	N	Y	N	Y	N
Student-Teacher Ratio Control	Y	N	Y	N	Y	N	Y	N
Number of observations	405	802	405	802	405	802	405	802
Clusters	106	103	106	103	106	103	106	103
Adjusted R^2	0.235	0.181	0.211	0.134	0.222	0.268	0.279	0.224

- Cannot be explained by changes in the schools that students attend or the composition of the private sector.
- Variance in test scores increases *within* schools.
- Robust to standard controls for peer effects.

Evidence on Mechanisms

	(1) Any Bonuses	(2) Bonuses for Exam Performance	(3) Bonuses for Teacher Attendance	(4) Share Getting Bonuses for Exams
Num. Pri. Schools	0.026	0.082***	-0.052**	0.037***
	(0.025)	(0.027)	(0.022)	(0.013)
Mean Dep. Var.	0.373	0.165	0.165	0.035
Number of observations	1,151	1,151	1,151	1,145
Clusters	108	108	108	108
Adjusted R^2	0.304	0.143	0.210	0.073

- Private schools become more likely to give teacher bonuses based on students' test scores instead of teacher attendance.

Model: Environment

- Students choose schools or the outside option to maximize their utility based on school characteristics, including quality.
  - Students differ in their responsiveness to schools' characteristics.
- Students' learning in a school depends on both vertical quality and instructional match.
  - Vertical quality: better facilities, better trained teacher, lower teacher absence.
  - Instructional match: a student who cannot do arithmetic does not benefit from learning calculus.
- Formally,
  - Poorer Students: optimal instructional level is 0 on the Hotelling line.
  - Richer Students: optimal instructional level is 1 on the Hotelling line.
- N schools choose their characteristics to maximize their profits.

Student's Problem

A student i of type z chooses a school j or not to enroll to maximize

$$u_{ijz} = \delta_z VA_{jz} + \Gamma_z X_{ij} + \xi_j + \epsilon_{ij},$$

where

$$VA_{j,poor} = v_j - \beta(0 - h_j)^2$$

and

$$VA_{j,rich} = v_j - \beta(1 - h_j)^2.$$

- h_j: school j's choice of instructional level on the unit line.
- X_{ij}: Characteristics of school j and student i.
- ξ_j: School quality unobserved to the econometrician.
- ϵ_{ij}: type 1 extreme value error.

School's Problem

A school chooses its instructional level $h_j \in [0, 1]$, its vertical quality v_j, and its price, fee_j, to maximize

$$\pi_j = fee_j \times s_j(v_j, h_j, fee_j, X_j, \xi_j) - c(v_j, h_j, fee_j, X_j, \xi_j).$$

- s_j: the number of students who attend school j.
- c: the cost of providing education, which depends on both the school's quality and the number of students.

How Can Competition Lead to Increased Inequality?

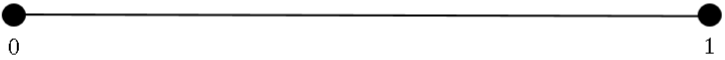
A simple example illustrates how competition can lead to increased inequality:

- Assume instructional match is the only characteristic that matters ($\Gamma_z = 0$, $\xi_j = 0$, $v_j = 0$) and that price is fixed.
- There is one free, non-private option that gives utility u_o.
- Equal number of rich and poor students.
- δ_{rich} is ∞: Richer students always maximize learning.
- $\delta_{poor} < \delta_{rich}$: Poorer students respond positively to expected learning.
- There are $N \in \{1, 2\}$ private schools.

Equilibrium With 1 School

Prediction

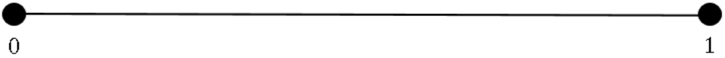
For N=1, there is a unique equilibrium where the school places at $h^* = 1 - (-u_o)^{1/2}$.



Equilibrium With 2 Schools

Prediction

For N=2, if δ_{poor} is sufficiently small, the unique equilibrium is $(1,1)$.



Key Intuition

- Spence distortion: oligopolistic schools respond to the marginal consumer rather than inframarginal consumers when they choose their quality.
- Advantaged students are more responsive to quality when they make their enrollment decisions.
- Increasing competition makes these students "more marginal."
- Schools' choices of quality will be very different from the social planner's since the social planner cares about inframarginal students.
- In other words, schools may respond to competitive incentives by competing to have the most advanced instructional level, reducing learning.

Mapping Model to Data

- To map the model to data, we need to measure both student types and school quality.
- First
  - Assign student types in two different data sets.
    - Child survey: contains test score data.
    - Household survey: contains distance to school and school choice data.
  - Estimate school quality.
- Then
  - Estimate the model.

Measuring School Quality

Use value-added methodology to estimate a type z's test scores y_{it} in a school j:

$$y_{it} = \tau_{gz,1} y_{i,t-1} + \tau_{gz,2} y_{i,t-1}^2 + \omega_{gz} + \alpha_{zt} + VA_{zjn} + \epsilon_{it}$$

where i is a student, g is a grade, z is a type, t is a year, and n is the number of competitors in the market place.

- VA_{zjn}: school by type by number of competitors fixed effect.
- $\tau_{gz,1}, \tau_{gz,2}$: grade by type specific coefficients.
- ω_{gz}: grade by type fixed effect.
- α_{zt}: type by year fixed effect.

Measuring Types in the (In-School) Child Survey

- Factor analysis of normalized HH assets and their interactions.
  - Predict the first factor.
- Separate students into those above and below a data-driven cut-off for the private school population.

Data-Driven Cut-off

Estimate

$$y_{it} = \tau_{gz,1} y_{i,t-1} + \tau_{gz,2} y_{i,t-1}^2 + \omega_{gz} + \alpha_{zt} + \phi_{jn} + \sum_{jn} \kappa_{jn} \mathbb{1}_i^{rich} + \epsilon_{it}$$

- Choose cut-off that maximizes F-statistic for κ_{jn}.
- Selects 55th percentile.

Types in the Household Data

- Child survey does not include distance measures.
- Household survey does not include test scores.
- To calculate types in household survey, take advantage of overlap in data.
- Lasso logistic regression of type in household survey on normalized HH assets for students in the overlapping sample.
- Predict probability of being an advantaged type p_H in full household survey.

Estimating the Model

Recall

$$u_{ijzt} = \delta_z VA_{jz} + \Gamma_z X_{ijt} + \xi_{jn} + \epsilon_{ijt},$$

where

$$VA_{j,dis} = v_j - \beta(0 - h_j)^2$$

and

$$VA_{j,ad} = v_j - \beta(1 - h_j)^2.$$

To estimate these parameters:

1. Estimate the determinants of school choice (demand side).
   - Find that $\delta_{poor} < \delta_{rich}$.
2. Estimate equilibrium choice of school quality (supply side).
   - Find that h_{jt} is increasing in the number of schools.
   - Estimate β.

Demand Estimation

To estimate the parameters of the utility function

$$u_{ijzt} = \delta_z VA_{jzn} + \Gamma_z X_{ijt} + \xi_{jn} + \epsilon_{ijt},$$

first, define

$$\zeta_{jn} = \xi_{jn} + \Gamma_z^{school} X_{jn}^{school}.$$

Then, estimate $\{\Gamma_z^{indiv}, \delta_{poor}, \delta_{rich}, \zeta_{jn}\}$ by maximizing

$$\sum_{ijt} \mathbb{1}_{ijt} log(p_{ijt}),$$

where

$$p_{ijt} = P(type_i = rich) \frac{e^{\delta_{rich} VA_{j,rich,n} + \Gamma_{rich}^{indiv} X_{ijt}^{indiv} + \zeta_{jn}}}{\sum_k e^{\delta_{rich} VA_{k,rich,n} + \Gamma_{rich}^{indiv} X_{ikt}^{indiv} + \zeta_{kn}}} + (1 - P(type_i = rich)) \frac{e^{\delta_{poor} VA_{j,poor,n} + \Gamma_{poor}^{indiv} X_{ijt}^{poor} + \zeta_{jn}}}{\sum_k e^{\delta_{poor} VA_{k,poor,n} + \Gamma_{poor}^{indiv} X_{ikt}^{indiv} + \zeta_{kn}}}.$$

▸ Identifying Γ_z

Demand Estimation Results

	(1) Coefficient	(2) Se
$distance_{ij} \times \mathbb{1}_{poor}$	-1.447***	0.076
$distance_{ij} \times \mathbb{1}_{rich}$	-0.393***	0.050
$VA_{j,poor,n} \times \mathbb{1}_{poor}$	0.520*	0.283
$VA_{j,rich,n} \times \mathbb{1}_{rich}$	1.218***	0.215
$fee_{jn} \times \mathbb{1}_{rich}$	0.212*	0.114
fee_{jn}	-4.521***	0.197

- Do the demand estimates make intuitive sense?

Descriptive Evidence

Supply Estimation: Equilibrium Choice of Horizontal Quality

Recall

$$\pi_j = fee_j \times s_j(v_j, h_j, fee_j, X_j, \xi_j) - c(v_j, h_j, fee_j, X_j, \xi_j),$$

so

$$\frac{\partial \pi_j}{\partial h_j^*} = \left(fee_j - \frac{\partial c}{\partial s_j} \right) \frac{\partial s_j}{\partial h_j^*} - \frac{\partial c}{\partial h_j^*} = 0.$$

Equilibrium Choice of Horizontal Quality

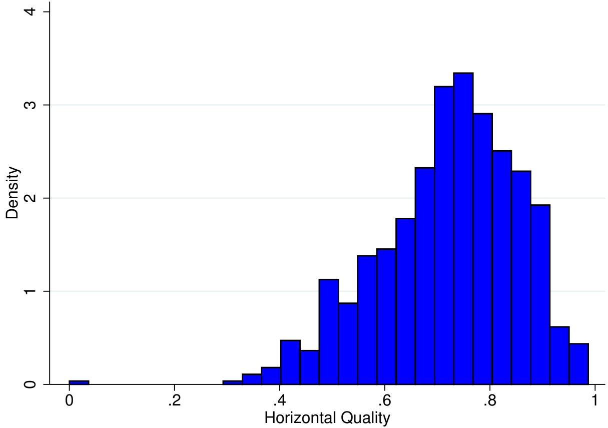
If we assume $\frac{\partial c}{\partial h_j^*} = 0$ and c is weakly-concave in share of students s, profit-maximization implies that

$$\frac{\partial s_{jt}}{\partial h_{jt}} = 0.$$

- Assume we observe schools choosing their equilibrium characteristics (fees, vertical quality) in the data.
- Then, we can solve for what equilibrium horizontal quality a private school should choose given its own characteristics and the characteristics of the other schools in the market.

Identifying Expression

Estimates of Equilibrium Horizontal Quality



Average horizontal quality is 0.7, even though 55% of private school students have an optimal instructional level of 0.

Interpreting Estimates

- Spence distortions are important and lead private schools to compete more intensively for wealthy students.
- Driven by the fact $\delta_{rich} > \delta_{poor}$.
- From the point of view of a social planner who wants to maximize learning, instructional level is misallocated.
- h_{jt} is positively correlated with number of private schools, consistent with motivating results.

Identifying β

Using the fact that

$$VA_{j,poor} = v_j - \beta h_j^2$$

and

$$VA_{j,rich} = v_j - \beta(1 - h_j)^2,$$

identify β from

$$\widehat{VA_{j,rich,n}} - \widehat{VA_{j,poor,n}} = \beta(2h_{jt} - 1) + \epsilon_{jt}.$$

Key intuition: the relationship between schools' optimal choices of h_{jt} and the difference between school quality for disadvantaged and advantaged students identifies β.

Importance of Horizontal Quality

Recall:

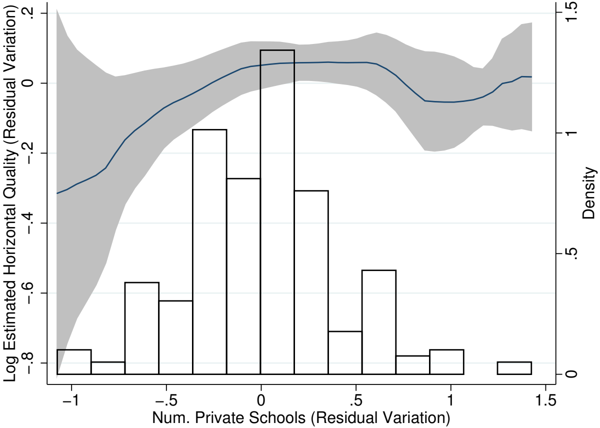
$$VA_{j,poor} = v_j - \beta h_j^2$$

and

$$VA_{j,rich} = v_j - \beta(1 - h_j)^2.$$

- β captures the relative importance of instructional match for learning.
- Estimate $\beta = 0.36$.
- Interpretation: moving a school's instructional level from 1 to 0 increases a disadvantaged student's mean test score by 0.36 sd.

Horizontal Quality and Number of Schools



Delivers a prediction that we can test with our reduced-form exit-entry identification strategy.

Testing the Model Predictions

Test whether increased competition increases inequality between rich and poor students in the same private school:

$$\begin{aligned} y_{it} =& \rho_0 + \rho_1 num\_pri_{vt} + \rho_2 num\_pri_{vt} \times \mathbb{1}_{rich} + \eta_{zj} + \alpha_{zt} \\ &+ \omega_{gz} + \lambda_{gz} y_{i,t-1} + \phi_{gz} y^2_{i,t-1} + \epsilon_{it}, \end{aligned}$$

- num_pri_{vt}: number of private schools in village v and year t.
- $\mathbb{1}_{ad}$: indicator variable for advantaged.
- η_{zj}: school by type fixed effect.
- α_{zt}: type by year fixed effect.
- ω_{gz}: type by grade fixed effect.
- λ_{gz}, ϕ_{gz}: grade by type specific coefficient.

Potential confounders:

- Time trends.
- Compositional changes.

Exit and Entry of Private Schools

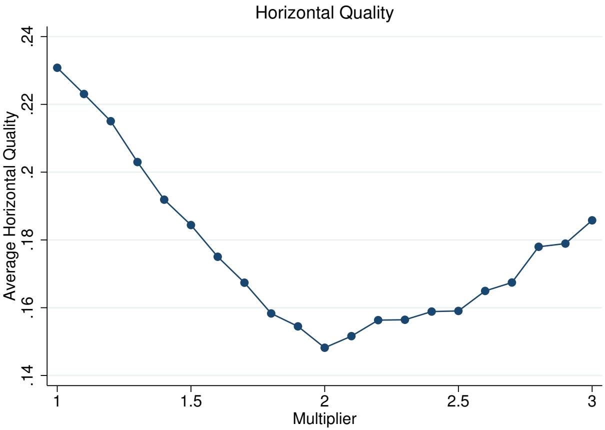
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Math		English		Urdu		Mean	
$\mathbb{1}_{rich} \times num\_pri_{vt}$	0.168***	0.145**	0.103**	0.093**	0.090*	0.066	0.121***	0.100**
	(0.055)	(0.056)	(0.043)	(0.045)	(0.046)	(0.048)	(0.038)	(0.040)
num_pri_{vt}	-0.132***	-0.137***	-0.066	-0.068	-0.033	-0.036	-0.073*	-0.077**
	(0.036)	(0.038)	(0.056)	(0.056)	(0.043)	(0.041)	(0.037)	(0.035)
Num. Gov School Control	Y	Y	Y	Y	Y	Y	Y	Y
Peer Controls	N	Y	N	Y	N	Y	N	Y
Student-Teacher Ratio Controls	N	Y	N	Y	N	Y	N	Y
Lagged Test Score Controls	Y	Y	Y	Y	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y	Y	Y	Y	Y
Year-Type-Report Card FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	6,794	6,760	6,794	6,760	6,794	6,760	6,794	6,760
Clusters	108	108	108	108	108	108	108	108
Adjusted R^2	0.577	0.590	0.606	0.611	0.634	0.644	0.690	0.707

Counterfactuals

1. Calculate the h_{jt} chosen by a social planner who wants to maximize average learning:
   - Average horizontal quality is .4.
   - Small test score increases and larger ($>$20%) decline in inequality.
2. Calculate the effects of choosing learning-maximizing h_{jt} with improved sorting.
   - Replace coefficients on value-added for rich and poor with $m \times \delta_{rich}$ for varying values of m.
   - Get improvements relative to case where sorting improves but school quality is unchanged to isolate gains from changing match-specific quality.

Counterfactual #2

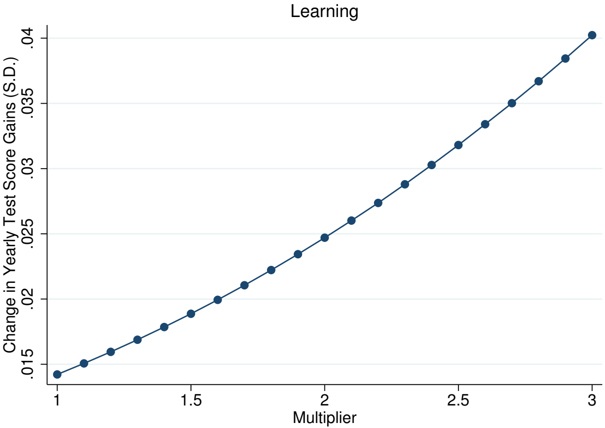
Changes in average value of h_{jt}



Why is this U-Shaped?

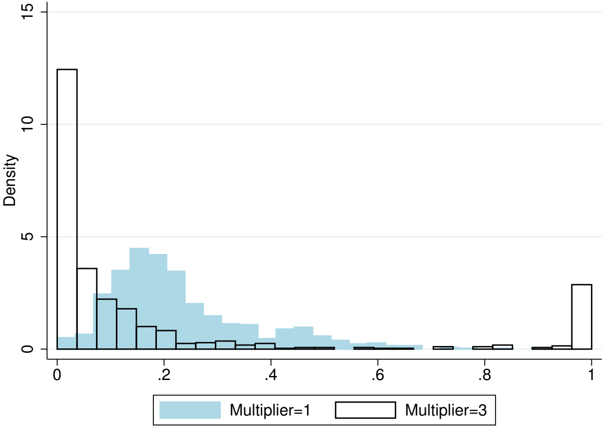
Counterfactual #2

Improvements in learning



Counterfactual #2

Improved sorting allows for product differentiation:



Conclusion

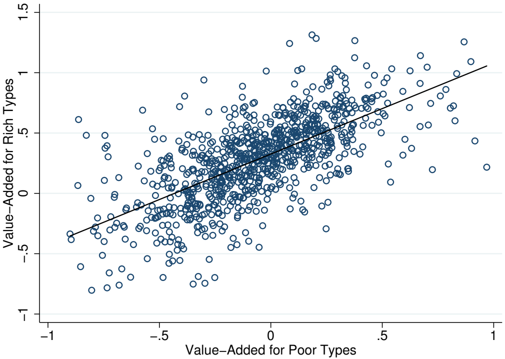
- Instructional match has a large effect on learning.
- Substantial misallocation of instructional levels in rural Pakistan, a context much like other low-income countries.
- Competitive incentives mean that virtually all private schools compete to be the best school for richer students, providing the "wrong" type of instruction.
- Improving instructional match yields the largest benefits when students are more responsive to quality when they sort into schools.
  - What's the policy implication?

Drivers of Exit and Entry

	(1) 1981 Population	(2) 1998 Population	(3) % Pop. Change	(4) Mean Assets	(5) Percent Own Land	(6) Gini Coefficient
num_pri_{vt}	248.879**	373.904*	0.009	0.017	-0.001	0.001
	(108.864)	(210.127)	(0.025)	(0.036)	(0.010)	(0.008)
Year FE	Y	Y	Y	Y	Y	Y
Village FE	N	N	N	Y	Y	Y
Number of observations	448	448	436	448	336	448
Clusters	112	112	109	112	112	112
Adjusted R^2	0.146	0.119	-0.005	0.964	0.841	0.265

Back

Relationship Between Type-Specific Value-Added Estimates



▸ Back

Correlation Between Exit and Entry and Village Trends

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Year FE	Y	Y	Y	Y	Y	Y
Village FE	N	N	N	Y	Y	Y
Number of observations	448	448	436	448	336	448
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Back

Demand Estimation: Step 2

Given estimates of ζ_{jn}, use the relationship

$$\zeta_{jn} = \xi_{jn} + \Gamma_z^{school} X_{jn}^{school}$$

and GMM to estimate Γ_z^{school} using residual variation in teacher salaries as an instrument for school fees. Back

Identifying Equilibrium Choice of Horizontal Quality

Divide $\frac{\partial s_{jt}}{\partial h_{jt}}$ by β to get an identifying equation for each jt

$$\sum_{it} P(type_i = ad)\delta_{ad}(1 - h_{jt})p_{ij,ad,t}(1 - p_{ij,ad,t}) + (1 - P(type_i = ad))\delta_{dis}h_{jt}(p^2_{ij,dis,t} - p_{ij,dis,t}) = 0$$

that does not contain any other unknowns but h_{jt}. Back

Robustness: Peer and Class-size Controls

	(1) Math	(2) English	(3) Urdu	(4) Mean
$\mathbb{1}_{ad} \times num\_pri_{vt}$	0.102*	0.078*	0.055	0.083**
	(0.057)	(0.044)	(0.042)	(0.036)
num_pri_{vt}	-0.106**	-0.054	-0.029	-0.064**
	(0.036)	(0.044)	(0.035)	(0.027)
Peer Controls	Y	Y	Y	Y
Student-Teacher Ratio Controls	Y	Y	Y	Y
Lagged Test Score Controls	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y
Number of observations	6,788	6,788	6,788	6,788
Clusters	106	106	106	106
Adjusted R^2	0.583	0.602	0.640	0.703

Back

Effects of Entry on Always-Private Students

	(1) Math	(2) English	(3) Urdu	(4) Mean
$\mathbb{1}_{ad} \times num\_pri_{vt}$	0.112*	0.101**	0.098*	0.112**
	(0.067)	(0.050)	(0.050)	(0.048)
num_pri_{vt}	-0.107**	-0.061	-0.057	-0.078**
	(0.042)	(0.058)	(0.031)	(0.034)
Peer Controls	Y	Y	Y	Y
Number of Private Schools Controls	Y	Y	Y	Y
Lagged Test Score Controls	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y
Number of observations	5,845	5,845	5,845	5,845
Clusters	108	108	108	108
Adjusted R^2	0.597	0.609	0.652	0.715

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Descriptive Results on Determinants of School Choice

	(1) Changed Schools	(2) Knows Teacher's Name	(3) Chose School for Distance	(4) Chose School for Quality	(5) Mean School VA
$P(type_i = ad)$	0.310***	0.261***	-0.162	0.574***	0.153*
	(0.059)	(0.048)	(0.104)	(0.090)	(0.081)
$rank_{ij}$					0.050***
					(0.010)
$P(type_i = ad) \times rank_{ij}$					0.043*
					(0.023)
Mean	0.347	0.532	0.427	0.210	0.022
Observation Level	Child	Child-Year	Child	Child	Parent-School-Year
Number of observations	5,621	13,645	2,873	2,873	22,826
Clusters	1,694	1,695	1,153	1,153	684
Adjusted R^2	0.008	0.005	0.002	0.038	0.031

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